Theory of correlation transfer and correlation structure in recurrent networks

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Part I: a Pair of Neurons

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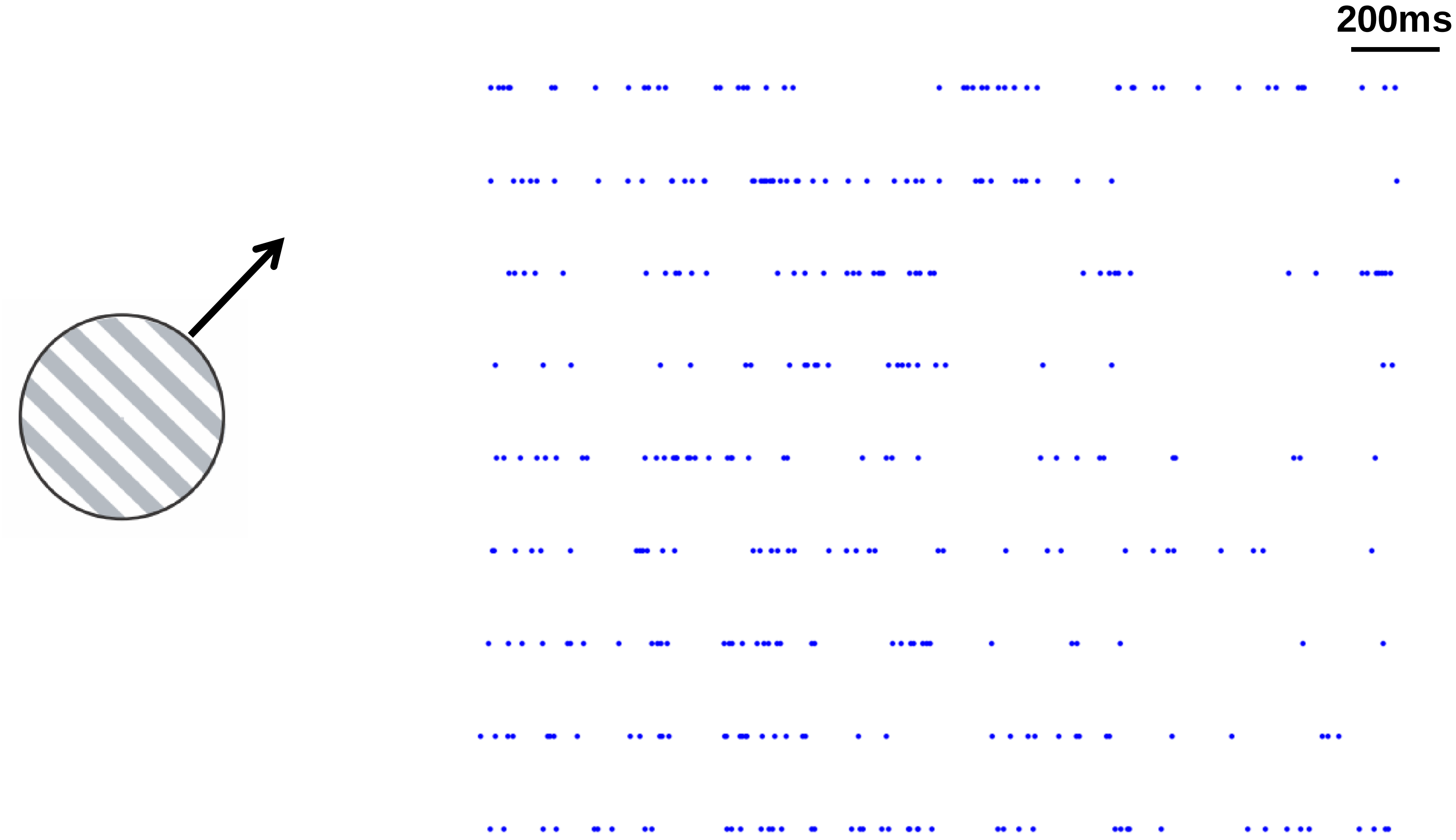
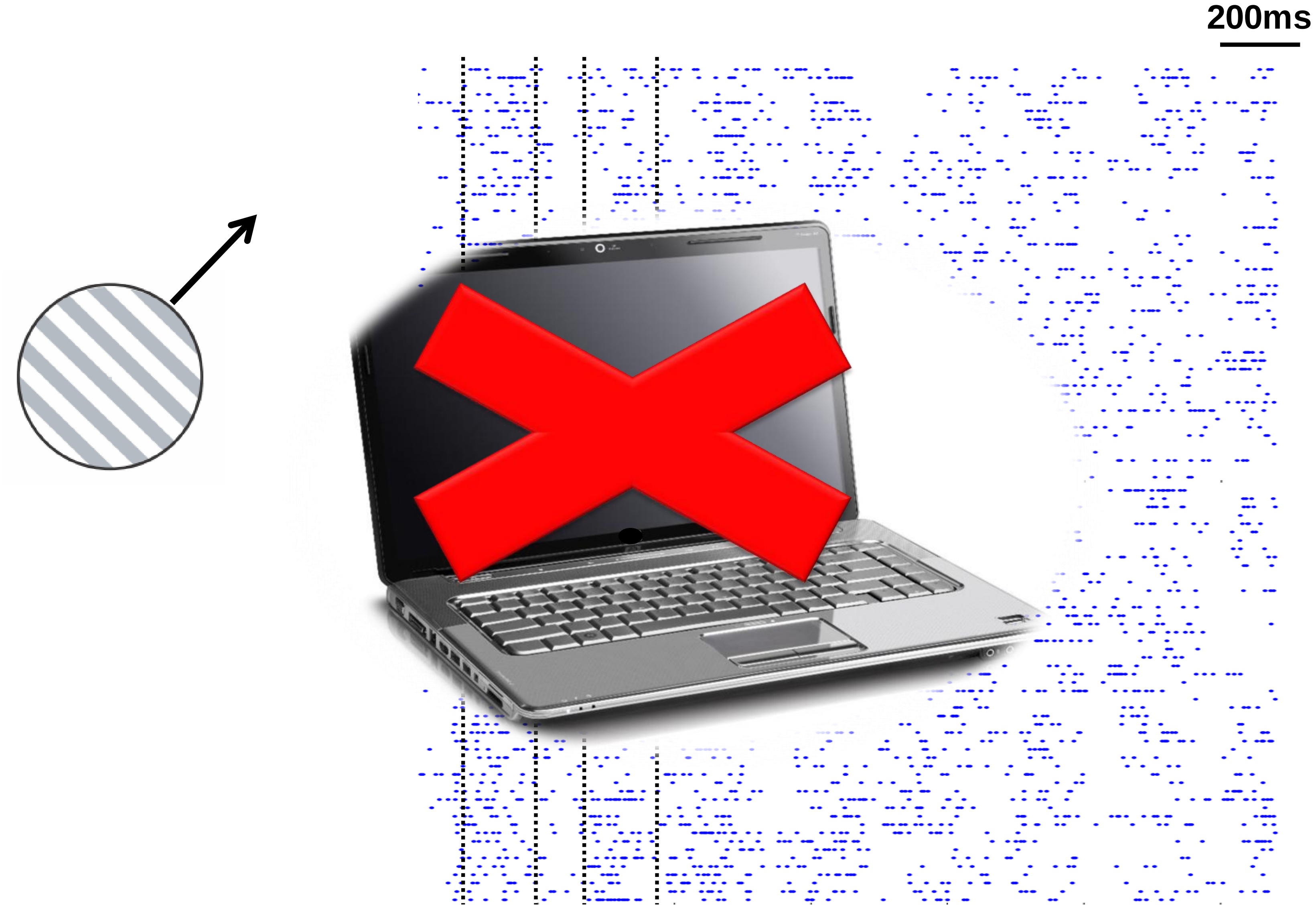
Cortical spiking variability:
non-reproducible spike trains



Cortical spiking variability: non-reproducible spike trains



Cortical spiking variability:
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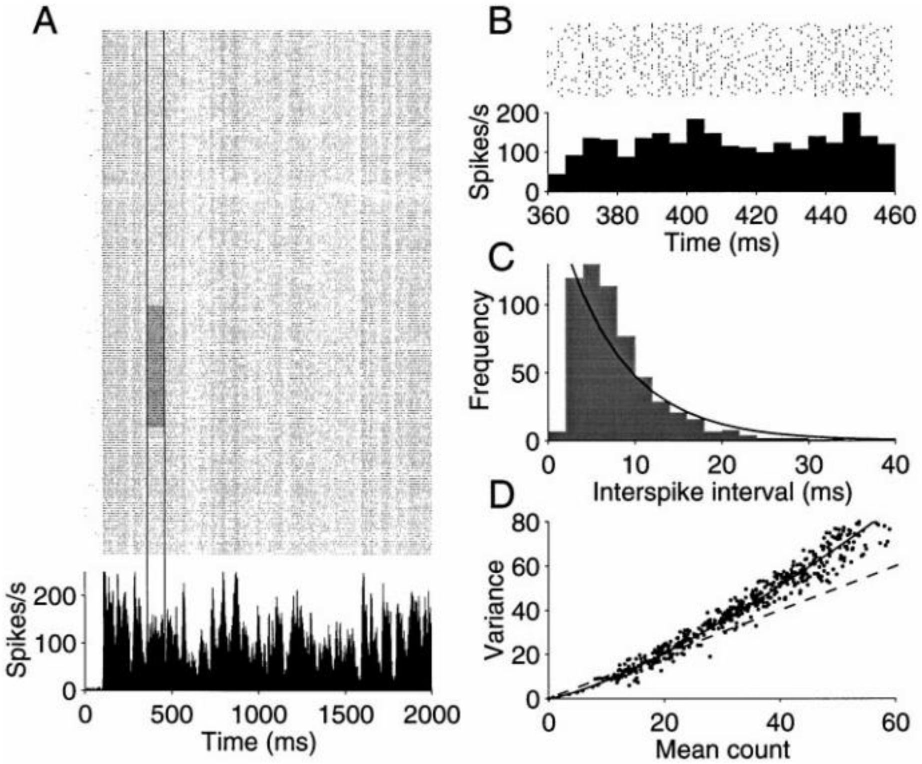
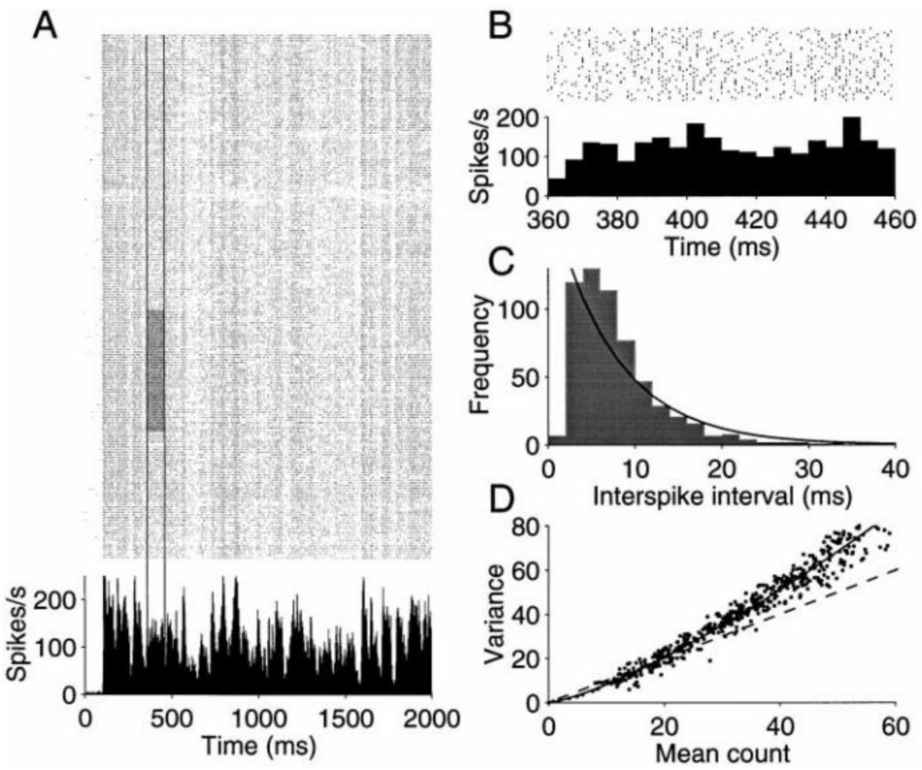


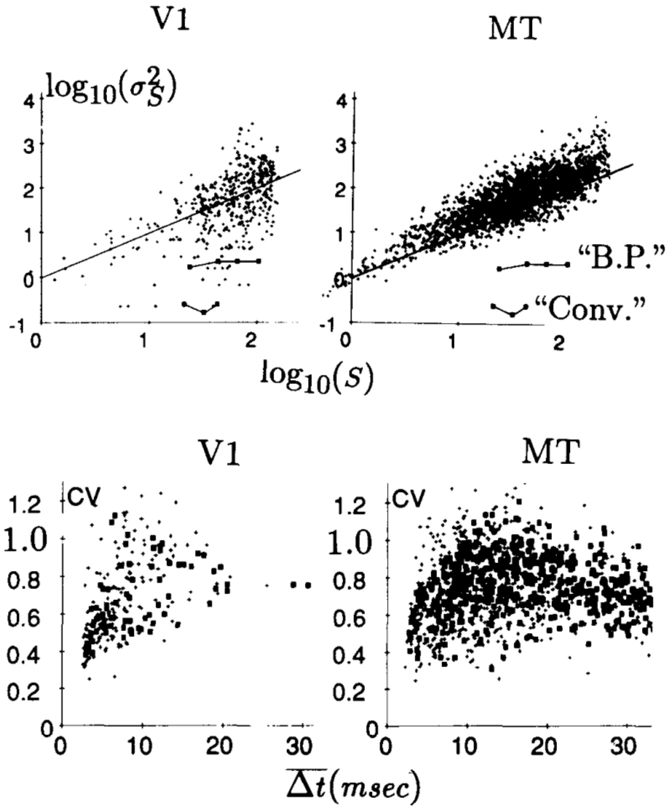
Shadlen and Newsome, 1998

Fano factor, F = Var(N) / <N> ~ 1.2

Cortical spiking variability: non-reproducible spike trains



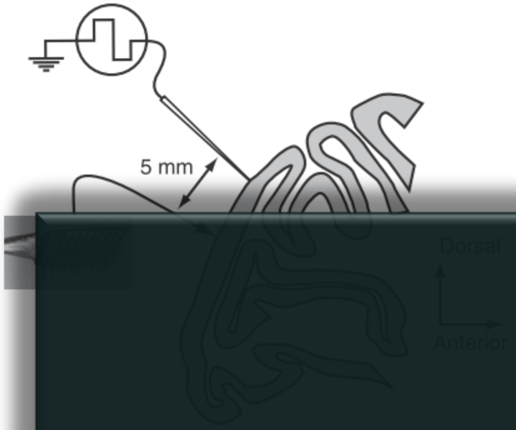
Shadlen and Newsome, 1998



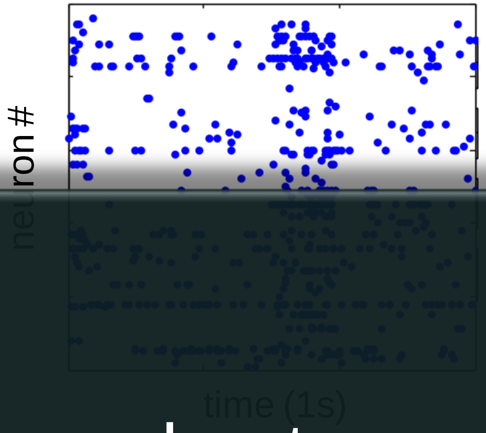
Softky and Koch, 1993

Fano factor, $F = \mathrm{Var}(N) / \langle N \rangle \sim 1.2$

Correlated activity

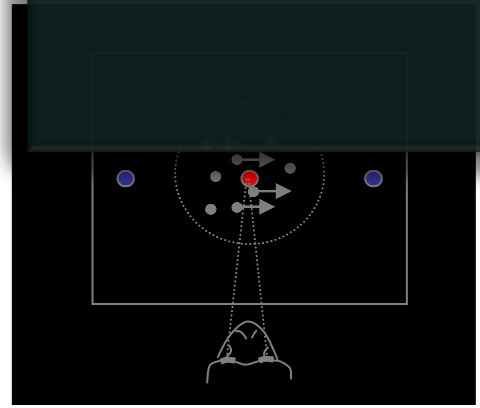


pair or multielectrode recordings

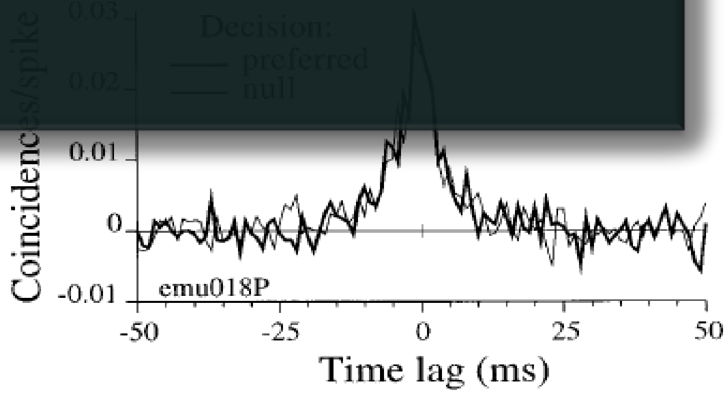


population activity

Why should we care about variability and correlations?



monkey behavior



peaks in CCFs: temporal coincidences

This is why you should care

- variability and correlations set fundamental limits of how much information can be extracted from the neuronal responses

Zohary et al, *Nature*, 1994

- how the observed variability and correlations arise from the underlying neuronal dynamics is largely unknown

Ginzburg and Sompolinsky, *Phys. Review E*, 1994
Moreno-Bote and Parga, *Phys. Review Letters*, 2006
de la Rocha et al, *Nature*, 2007
Kriener et al, *N. Computation*, 2008
Kumar et al, *N. Computation,* 2008
Renart et al, *Science*, 2010
Hertz, *N. Computation*, 2010

This is why you should care

- correlations open the door to estimate functional connectivity between neurons

Aertsen et al, *J. Neurophys*, 1989
Schneidman et al, *Nature*, 2006
Pillow et al, *Nature*, 2008
Cocco et al, *PNAS*, 2009

- variability and correlations might indicate the type of neuronal computations carried out by neuronal circuits

Abeles, Book: Corticonics, 1991
Softky, *Current Opi. Neurobiology*, 1995
Shadlen and Newsome, *J. of Neurosci.*, 1998
Diesmann et al, *Nature*, 1999

Outline

- Information limits set by neuronal correlations (an example)
- Firing rate and variability in LIF neurons with fast and slow synapses (FPE formalism and solutions)
- Correlation transfer in LIF neurons with fast and slow synapses (FPE and approximate solutions)
- Review of literature & main results about correlation transfer:

1. Neurons are sensitive to input correlations (strength and correlation time; Salinas and Sejnowski, *J. of Neurosci.*, 2000; Moreno-Bote et al, *Phys. Review Letters*, 2002)
2. Output correlation is lower than input correlation in spiking neurons (Moreno-Bote and Parga, *Phys. Review Letters*, 2006)
3. Firing rate and correlation coefficients are not independent (de la Rocha et al, *Nature*, 2007)

- Open questions

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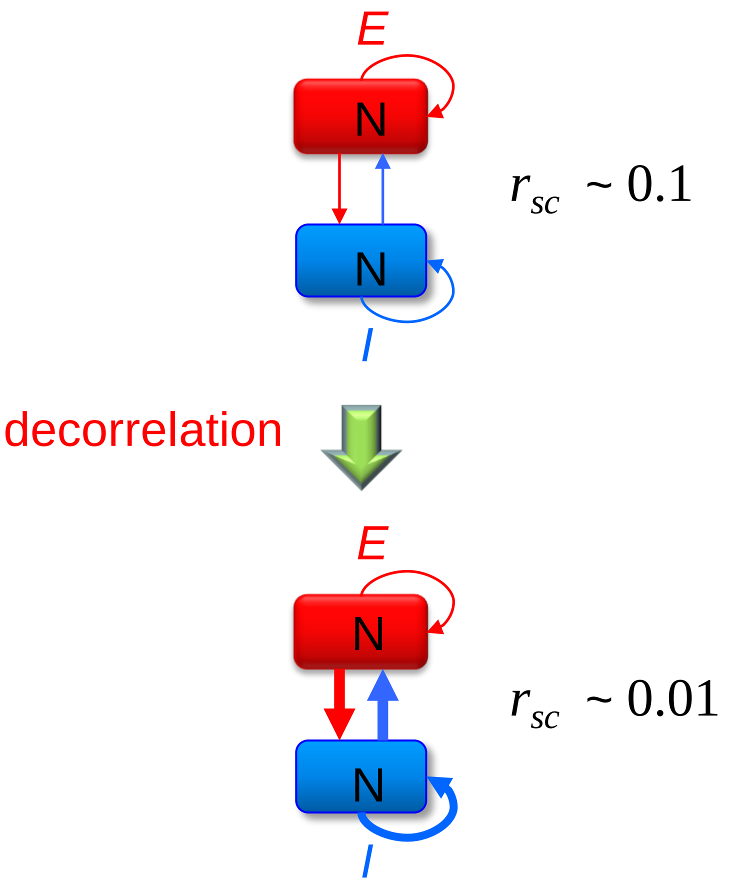
Signal/Noise limits induced by correlations

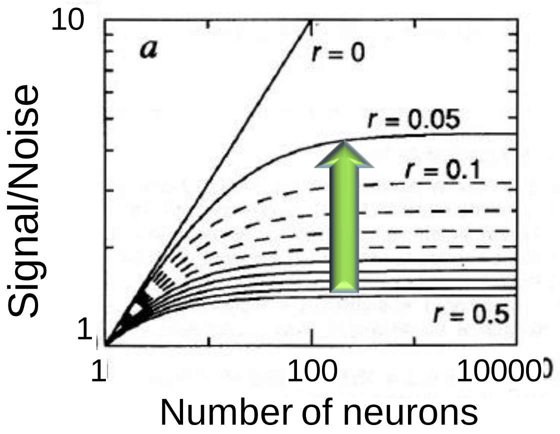


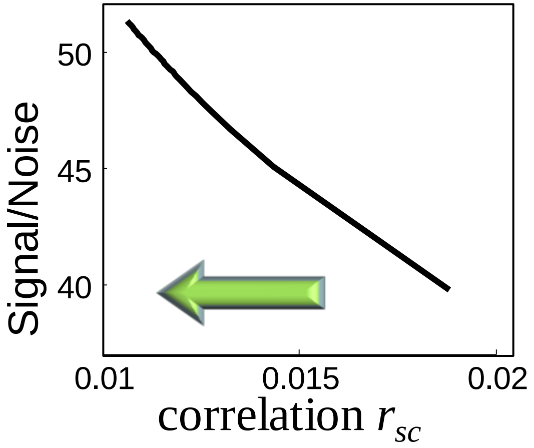
$r_{sc} \sim 0.1$

decorrelation

$r_{sc} \sim 0.01$

Zohary et al, 1994





- In homogenous neuronal populations, correlations are deleterious
- Whether it is possible to decorrelate while keeping firing rate and variability constant is under investigation

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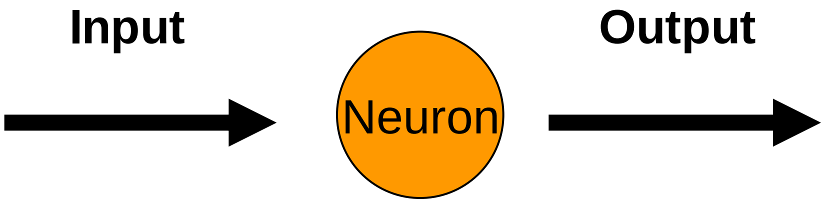
- Open questions

A Golden Problem:
Input-Output relationship

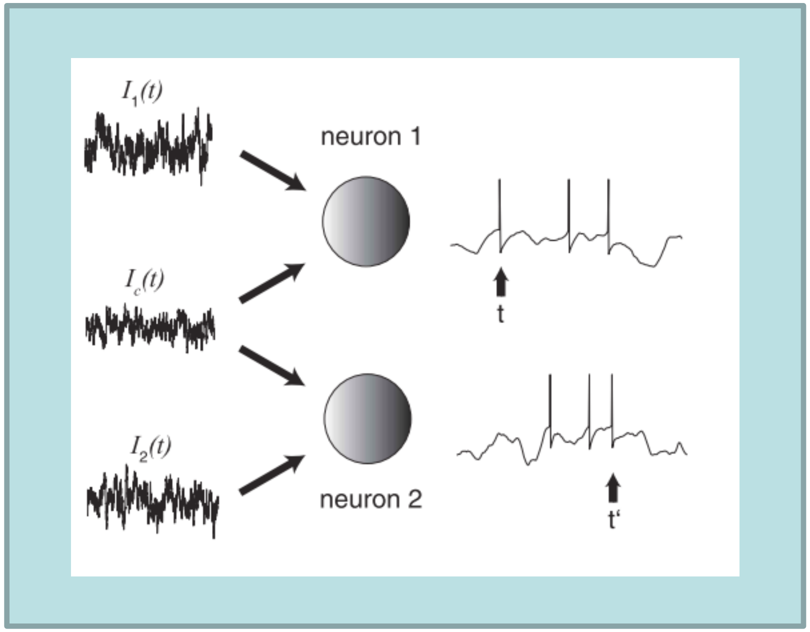


Input Neuron **Output**

The problem can be faced in the **statistical sense**: average quantities

ν_{in}	ν_{out}
$F_{N,in}$	$F_{N,out}$
ρ_{in}	ρ_{out}

A Golden Problem:
Input-Output relationship



Firing rate for a leaky integrate & fire (LIF) neuron with instantaneous synapses

$$\frac{dV(t)}{dt} = -\frac{V(t)}{\tau_m} + I(t),$$

$$I(t) = \sum_{j=1}^{C_E} J_{E_j}\, s_j(t) - \sum_{k=1}^{C_I} J_{I_k}\, s_k(t),$$

$$\nu^{-1} = \tau_{\mathrm{ref}} + \tau_m \sqrt{\pi} \int_{\frac{V_{\mathrm{res}}-\mu_V}{\sigma_V\sqrt{2}}}^{\frac{V_{\mathrm{th}}-\mu_V}{\sigma_V\sqrt{2}}} dx\, \mathrm{e}^{x^2}[1+\mathrm{erf}(x)]$$

$$\mathrm{CV}^2 = 2\pi\nu^2 \int_{\frac{V_{\mathrm{res}}-\mu_V}{\sigma_V\sqrt{2}}}^{\frac{V_{\mathrm{th}}-\mu_V}{\sigma_V\sqrt{2}}} dx\, \mathrm{e}^{x^2} \int_{-\infty}^{x} dy\, \mathrm{e}^{y^2}[1+\mathrm{erf}(y)]^2$$

$$\mu_V = \tau_m\mu; \quad \sigma_V^2 = \frac{1}{2}\tau_m\sigma^2$$

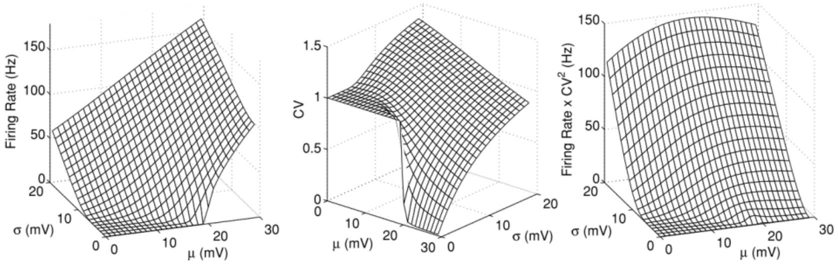


Figure 2: Mean firing rate ν (left), CV (middle), and product $\nu\mathrm{CV}^2$ (right) of the LIF neuron as a function of the mean and standard deviation of the depolarization. Parameters: $V_{\mathrm{th}} = 20$ mV, $V_{\mathrm{res}} = 10$ mV, $\tau_m = 10$ ms, and $\tau_{\mathrm{ref}} = 2$ ms.

Burkitt, Biol Cybern, 2006

Rate with non-instantaneous synapses
Fast neuronal dynamics

$$\dot{V} = -\frac{V}{\tau_m} + \mu + \frac{\sigma}{\sqrt{2\tau_s}} z(t)$$

$$\dot{z} = -\frac{z}{\tau_s} + \sqrt{\frac{2}{\tau_s}}\,\eta(t)$$

$$\longrightarrow$$

$$\left[\frac{\partial}{\partial x}(x - \epsilon z) + \epsilon^2 L_z\right] P = 0$$

$$\epsilon = \sqrt{\tau_m/\tau_s}$$

stationary FPE

$$L_z = \frac{\partial}{\partial z} z + \frac{\partial^2}{\partial^2 z}$$

In the long synaptic time scale limit $\tau_s \geq \tau_m$
we treat $\epsilon = \sqrt{\tau_m/\tau_s}$ as a small parameter

This limit is useful in the high conductance regime
(Destexhe et al.,Nat.Rev.Neurosc. 2003)
or when slow filters (NMDA, $\mathrm{GABA_B}$, etc) are important

firing rate

$$\nu = \int_{\hat{\Theta}/\epsilon}^{\infty} \frac{dz}{\sqrt{2\pi}}\, e^{-z^2/2}\, \nu_0(z)$$

$$\nu_0{}^{-1}(z) = \tau_m \ln\left(\frac{\hat{H} - \epsilon z}{\hat{\Theta} - \epsilon z}\right)$$

$$\hat{\Theta} = \sqrt{2}(\Theta - \mu\tau_m)/\sigma\sqrt{\tau_m} \text{ and } \hat{H} = \sqrt{2}(H - \mu\tau_m)/\sigma\sqrt{\tau_m}$$

Moreno-Bote and Parga, Phys Rev. Lett, 2004
Moreno-Bote and Parga, Neural Computation, 2010

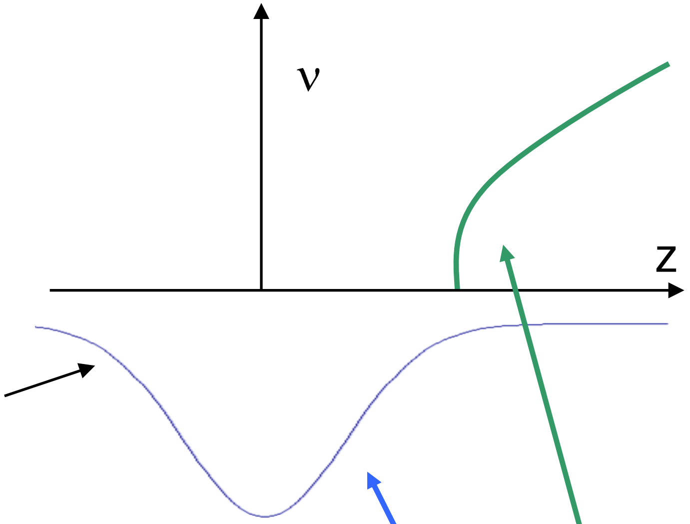
Rate with non-instantaneous synapses

At zero-th order

$$\frac{\partial}{\partial x}(x - \gamma z)P_0 = 0 \longrightarrow \nu(z) = \frac{1}{\tau_m log\left(\frac{\hat{H} - \epsilon z}{\hat{\Theta} - \epsilon z}\right)}$$

leak

constant drift



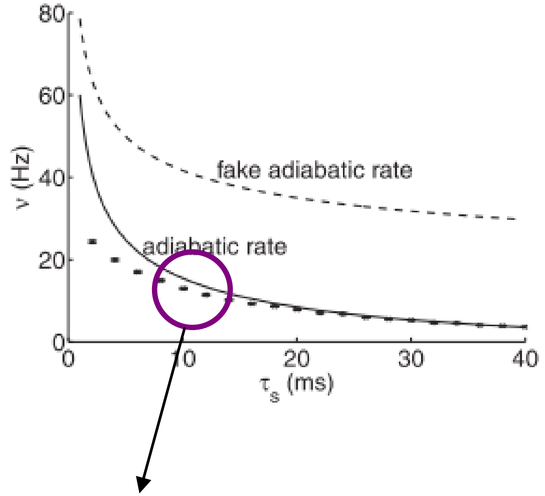
$$P(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}}$$

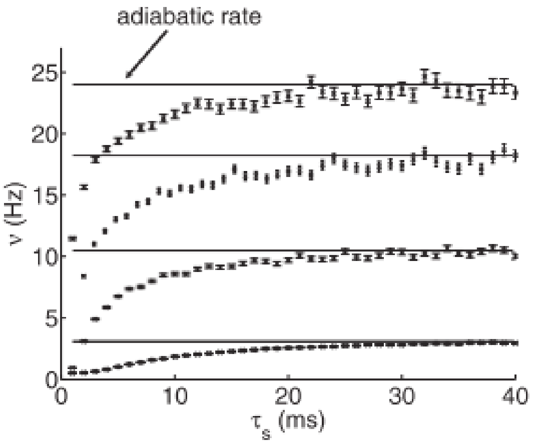
Firing rate

$$\nu = \int dz\, P(z)\, \nu(z)$$

The only approx. is $\tau_s \geq \tau_m$

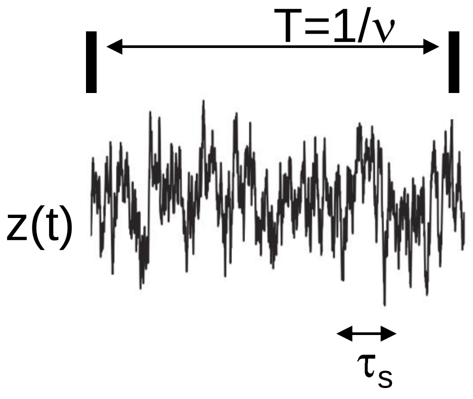
Rate with non-instantaneous synapses



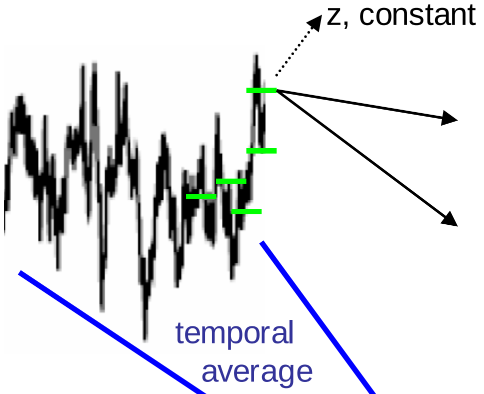


here $\tau_s = \tau_m = 10$ms

This is surprising because here z is not constant during an ISI of typical duration T = 100-200 ms.



Rate with non-instantaneous synapses



$$\tau_m \dot{x} = -x + \epsilon z$$

$$\nu(z) = \frac{1}{\tau_m log\left(\frac{\hat{H}-\epsilon z}{\hat{\Theta}-\epsilon z}\right)}$$

← instantaneous firing rate

$$\boxed{\nu = \int dz P(z)\, \nu(z)}$$

← firing rate

Why not $T = \int dz P(z)\, T(z)$?

It does not do a very good job

ISI for fixed z

Rate with non-instantaneous synapses
Fast synapses

In the short synaptic time scale limit $\tau_s \leq \tau_m$
we treat the inverse of $\epsilon = \sqrt{\tau_m/\tau_s}$ as a small parameter

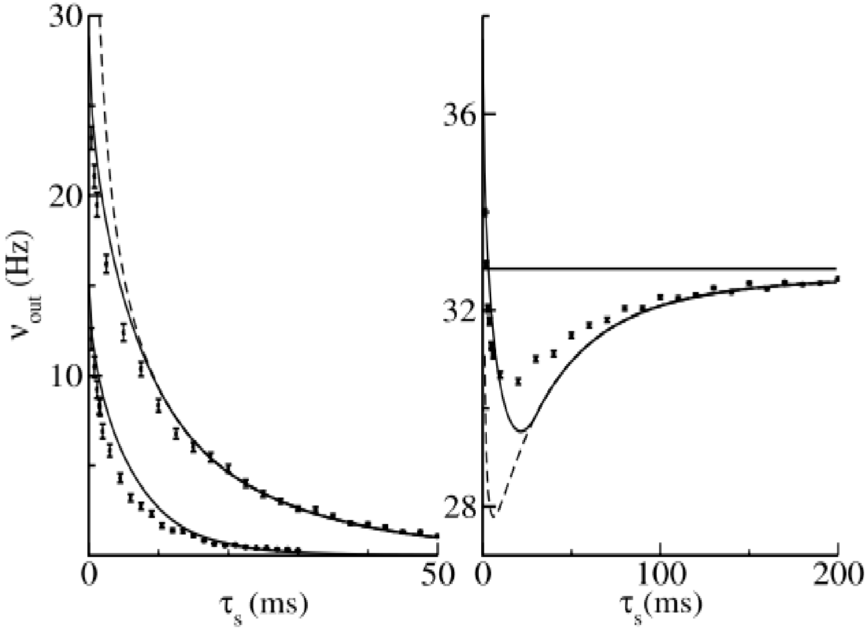
This limit is useful when AMPA receptors are abundant

firing rate

$$\frac{1}{\nu} = \tau_m\sqrt{\pi}\int_{\frac{V_r-\mu}{\sigma}+\frac{\alpha}{2}\sqrt{\frac{\tau_s}{\tau_m}}}^{\frac{\theta-\mu}{\sigma}+\frac{\alpha}{2}\sqrt{\frac{\tau_s}{\tau_m}}} \Psi(s)\,ds$$

$$\Psi(s) = e^{s^2}\left(1 + \mathrm{erf}(s)\right)$$

$$\alpha = \sqrt{2}\left|\zeta\left(\frac{1}{2}\right)\right| \simeq \sqrt{2}\ 1.46$$



Interpolating the fast and slow synaptic time scale limits

Brunel and Sergi, J theor Biol, 1998
Fourcaud and Brunel, Neural Comput., 2002
Moreno-Bote and Parga, Phys Rev Lett, 2004

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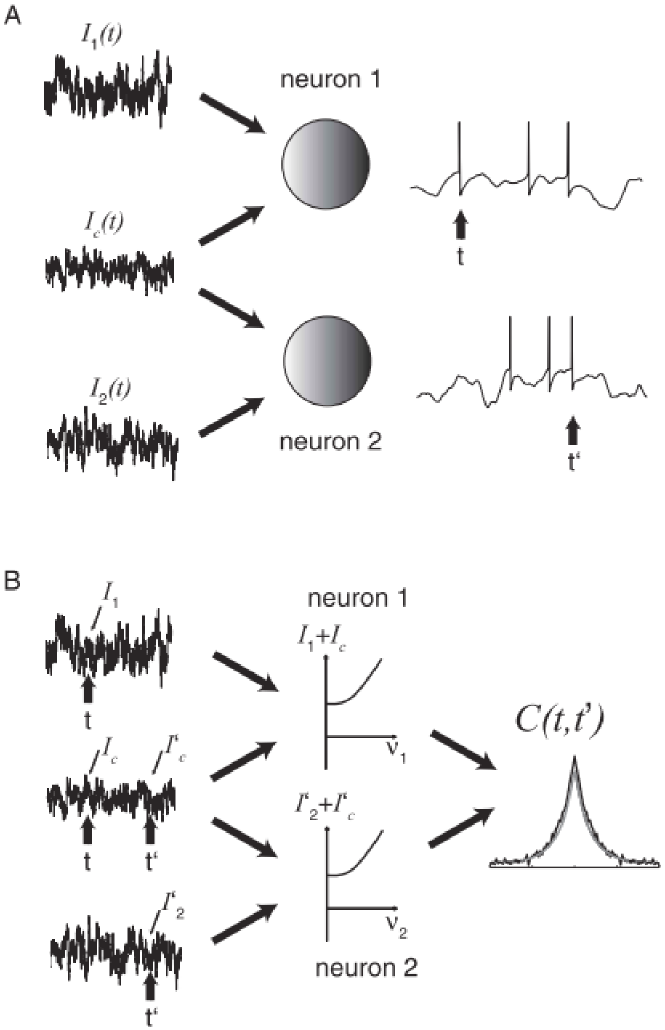
- Open questions

Correlations with non-instantaneous synapses



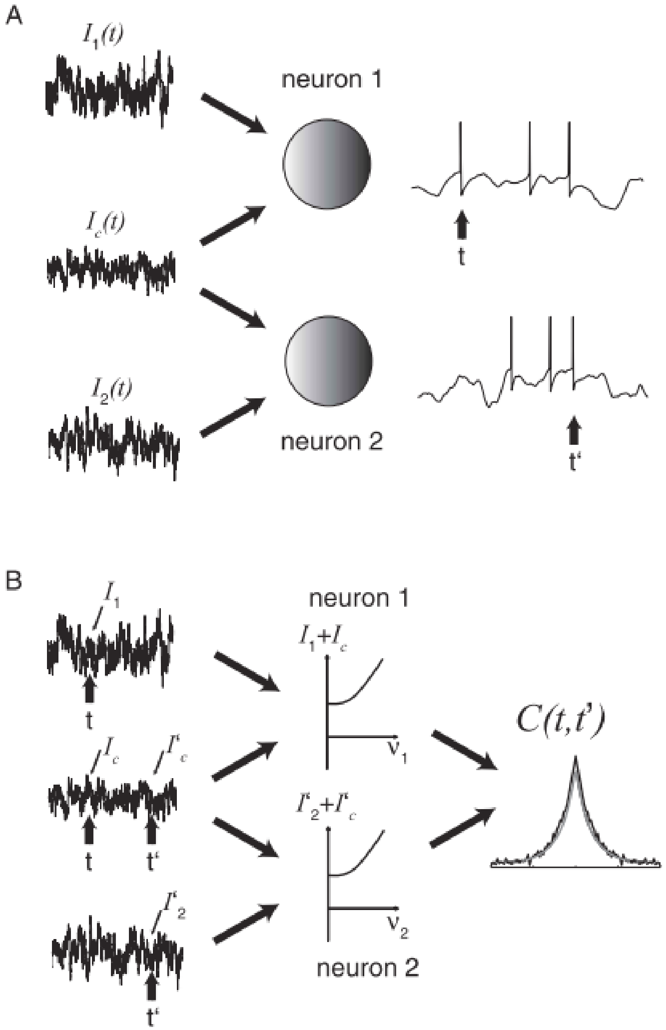
$$\tau_m \dot{V}_1 = -V_1 + \tau_m(I_1(t) + I_c(t)),$$
$$\tau_m \dot{V}_2 = -V_2 + \tau_m(I_2(t) + I_c(t)).$$

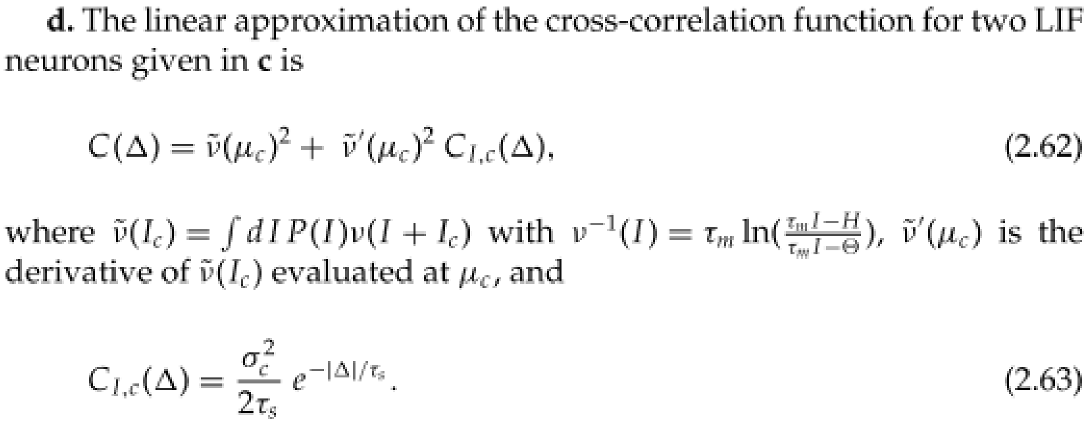
$$\tau_s \dot{I}_i(t) = -I_i(t) + \mu_{ind} + \sigma_{ind}\,\eta_i(t),$$
$$\tau_s \dot{I}_c(t) = -I_c(t) + \mu_c + \sigma_c\,\eta_c(t),$$

$$C(t,t') \simeq \int dI_1\, dI'_2\, dI_c\, dI'_c\; P(I_c, t; I'_c, t')\; P(I_1)\; P(I'_2)$$
$$\times\, \nu_1(I_1 + I_c)\; \nu_2(I'_2 + I'_c).$$

Moreno-Bote and Parga, Phys Rew Lett, 2006
Moreno-Bote and Parga, Neural Comput, 2010

Correlations with non-instantaneous synapses





d. The linear approximation of the cross-correlation function for two LIF neurons given in **c** is

$$C(\Delta) = \tilde{\nu}(\mu_c)^2 + \tilde{\nu}'(\mu_c)^2\, C_{I,c}(\Delta), \tag{2.62}$$

where $\tilde{\nu}(I_c) = \int dI\, P(I)\nu(I + I_c)$ with $\nu^{-1}(I) = \tau_m \ln(\frac{\tau_m I - H}{\tau_m I - \Theta})$, $\tilde{\nu}'(\mu_c)$ is the derivative of $\tilde{\nu}(I_c)$ evaluated at μ_c, and

$$C_{I,c}(\Delta) = \frac{\sigma_c^2}{2\tau_s}\, e^{-|\Delta|/\tau_s}. \tag{2.63}$$

Moreno-Bote and Parga, Phys Rew Lett, 2006
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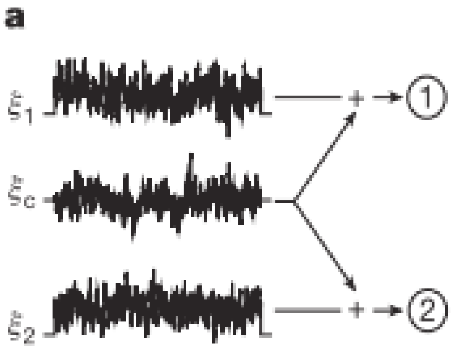
Correlations with instantaneous synapses



$$\tau_m \frac{dV_i}{dt} = -V_i + \mu + \sigma\sqrt{\tau_m}\left(\sqrt{1-c}\,\xi_i(t) + \sqrt{c}\,\xi_c(t)\right)$$

$$I_i = \mu_i + \sigma_i\left(\sqrt{1-c}\,\xi_i(t) + \sqrt{c}\,\xi_c(t)\right)$$

$$\rho \approx S(\mu,\sigma)c = \frac{\sigma^2\left(\frac{dv}{d\mu}\right)^2}{CV^2 v}c$$

de la Rocha et al, Nature, 2007

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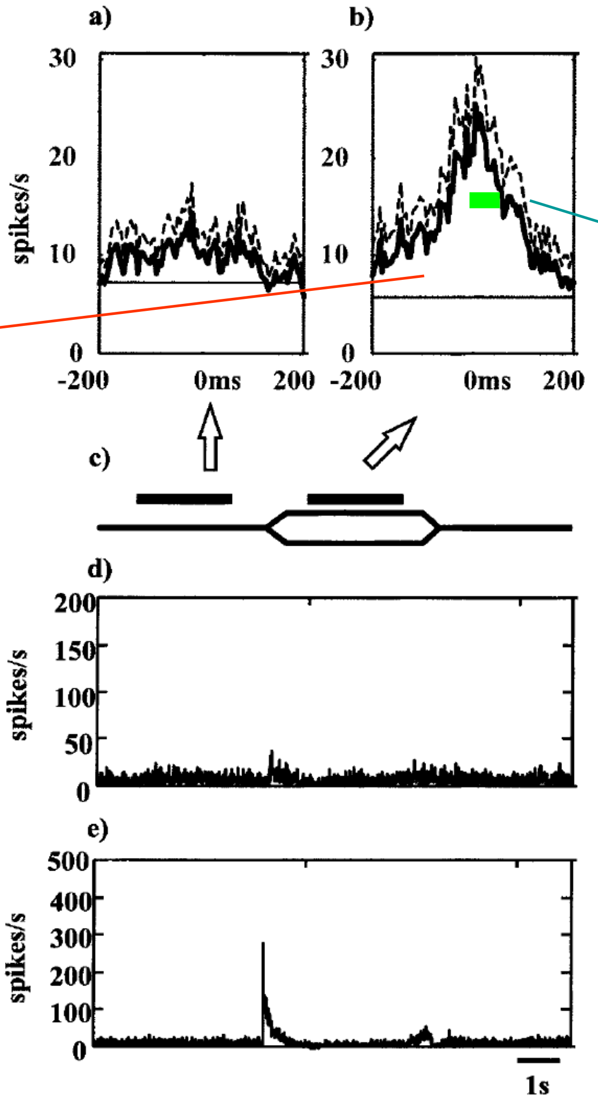
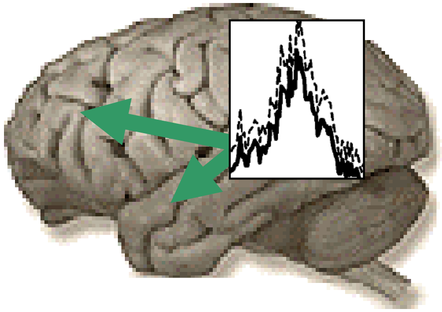
- Open questions

I. Correlated activity in primary auditory cortex



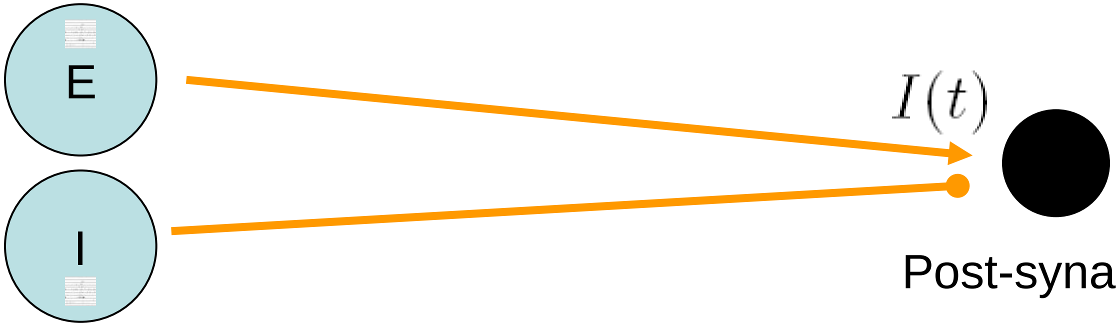
15 ms

Exponential-like correlations



deCharms and Merzenich, 1996

I. Model. The total presynaptic current



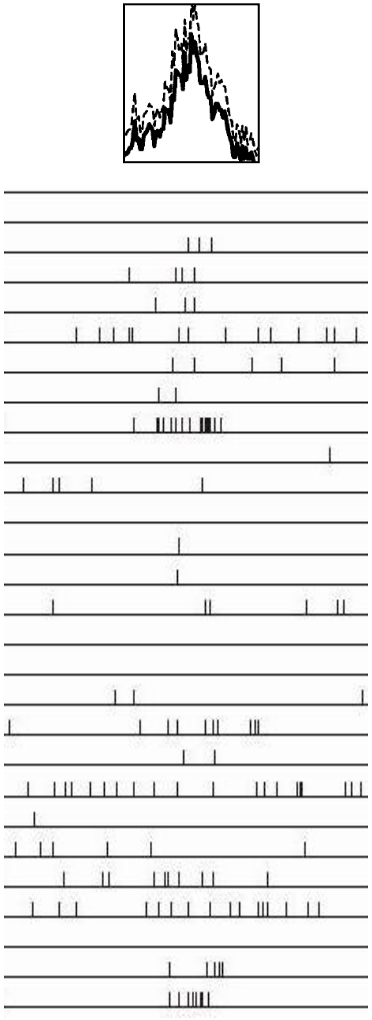
Post-synaptic Neuron

$$\dot{V}(t) = -\frac{V(t)}{\tau_m} + I(t)$$

Leaky Integrate-and-Fire neuron

$$I(t) = J_E \sum_{i=1}^{N_E} \sum_{k} \delta(t - t_i^k) - J_I \sum_{j=1}^{N_I} \sum_{l} \delta(t - t_j^l)$$

I. Model. Temporal Correlations



i, p=E,I

j, q=E,I

Auto-correlations:

$$C_p(t,t') \equiv \left\langle \left(\sum_k \delta(t - t_i^k) - \nu_p \right) \left(\sum_{k'} \delta(t' - t_i^{k'}) - \nu_p \right) \right\rangle$$

$$= \nu_p \delta(t-t') + \nu_p \left(\frac{F_p - 1}{2\tau_c} \right) e^{-\frac{|t-t'|}{\tau_c}}$$

rate

Fano factor

Cross-correlations:

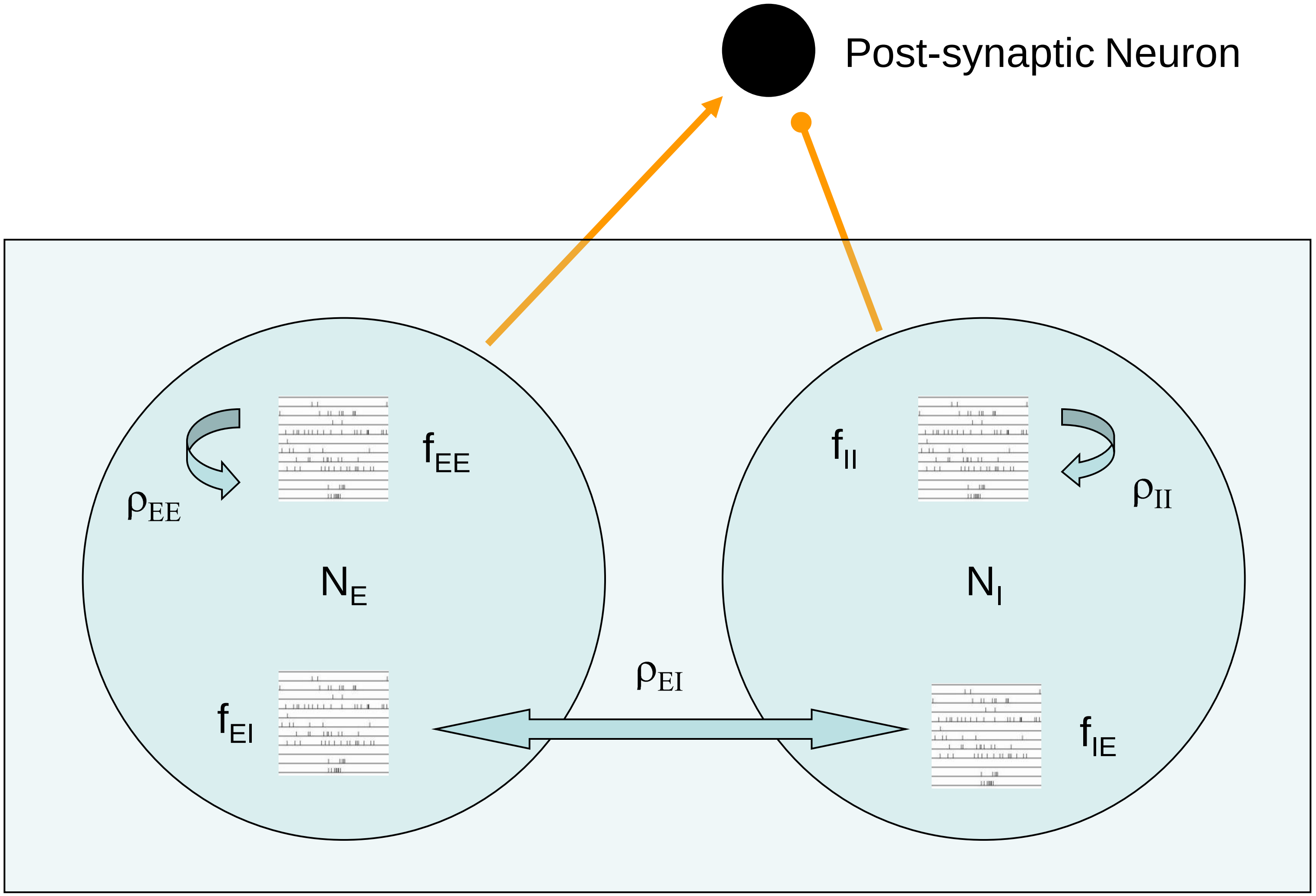
$$C_{pq}(t,t') \equiv \left\langle \left(\sum_{k_p} \delta(t - t_i^{k_p}) - \nu_p \right) \left(\sum_{k_q} \delta(t' - t_j^{k_p}) - \nu_q \right) \right\rangle$$

corr. time

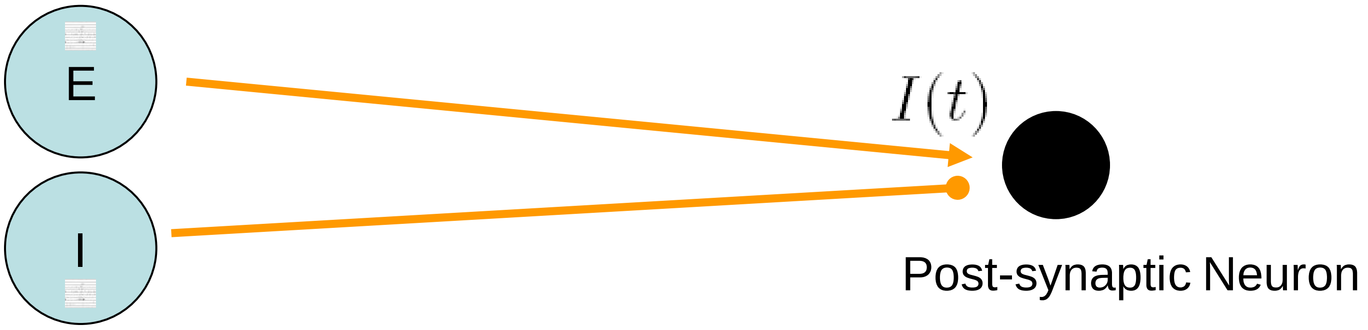
$$= \sqrt{\nu_p \nu_q} \left(\frac{\rho_{pq} \sqrt{F_p \, F_q}}{2\tau_c} \right) e^{-\frac{|t-t'|}{\tau_c}}$$

correlation coefficient

I. Model. Spatial Correlations



I. Results. Properties of the syn. current



$$I(t) = J_E \sum_{i=1}^{N_E} \sum_{k} \delta(t - t_i^k) - J_I \sum_{j=1}^{N_I} \sum_{l} \delta(t - t_j^l)$$

$$\mu = J_E N_E \nu_E - J_I N_I \nu_I$$

mean current

correlation magnitude

$$C_{current}(t, t') = \sigma_w^2 \left[\delta(t - t') + \frac{\alpha}{2\tau_c} e^{-\frac{|t-t'|}{\tau_c}} \right]$$

white noise variance

correlation time

I. Results. α and τ_c

$$C_{current}(t,t') = \sigma_w^2 \left[\delta(t-t') + \frac{\alpha}{2\tau_c}\, e^{-\frac{|t-t'|}{\tau_c}} \right]$$

correlation magnitude

correlation time

$$\begin{aligned}
\sigma_w^2 &= J_E^2 N_E \nu_E + J_I^2 N_I \nu_I \\
\alpha\ \sigma_w^2 &= J_E^2 N_E \nu_E [(F_E - 1) + f_{EE}(f_{EE} N_E - 1) F_E\ \rho_{EE}] \\
&\quad + J_I^2 N_I \nu_I [(F_I - 1) + f_{II}(f_{II} N_I - 1) F_I\ \rho_{II}] \\
&\quad - 2\ J_E J_I\ f_{EI} f_{IE} N_E N_I\ \sqrt{\nu_E \nu_I}\ \sqrt{F_E\ F_I \rho_{EI}}\ .
\end{aligned}$$

I. How to generate such a current?

$$I(t) = J_E \sum_{i=1}^{N_E} \sum_{k} \delta(t - t_i^k) - J_I \sum_{j=1}^{N_I} \sum_{l} \delta(t - t_j^l)$$

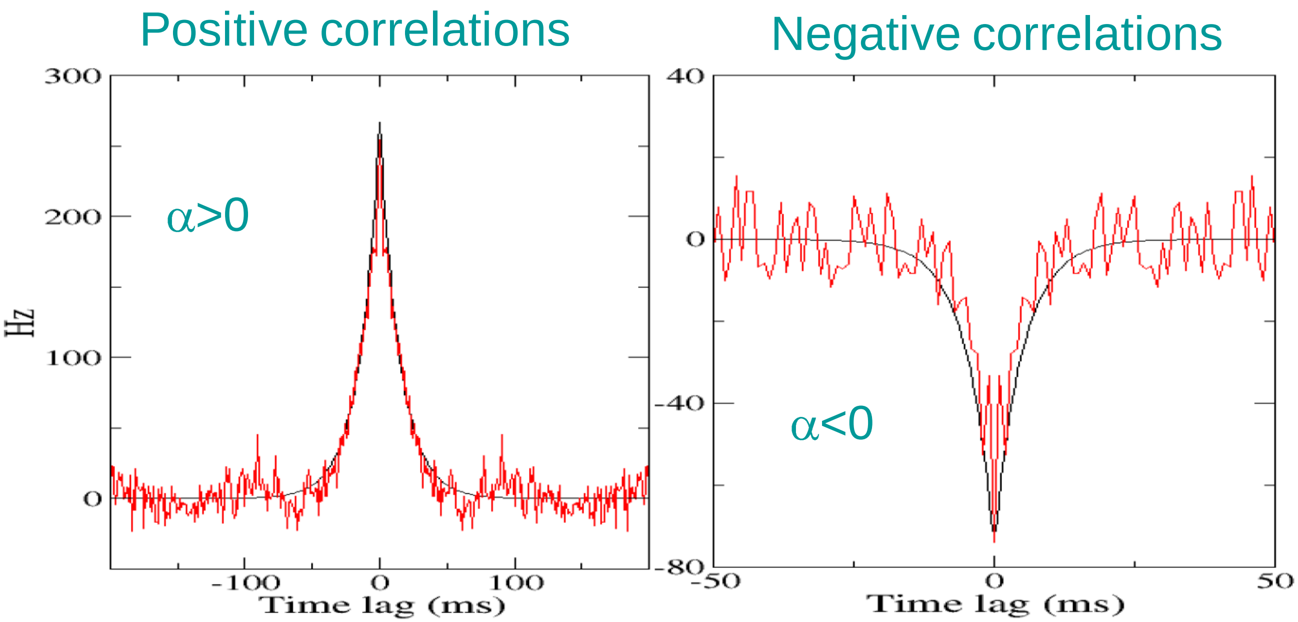
$$\mu = J_E N_E \nu_E - J_I N_I \nu_I$$

$$C_{current}(t, t') = \sigma_w^2 \left[\delta(t - t') + \frac{\alpha}{2\tau_c} \, e^{-\frac{|t-t'|}{\tau_c}} \right]$$

Why a simple representation of the current is required?

1. Generating the current in the way defined above is complex.
2. If the representation of the current is simple enough, it can allow us to find an analytical solution in some limits.
3. It can be used to simulate neurons receiving correlated inputs.
4. It can be used to stimulate real neurons with current waves mimicking correlated inputs.

I. Results. Generating I(t) using an auxiliar OUP


Positive correlations

Negative correlations

$$I(t) = \mu + \sigma_w \eta(t) + \sigma_w \frac{\beta}{\sqrt{2\tau_c}} z(t)$$

$$\dot{z}(t) = -\frac{z}{\tau_c} + \sqrt{\frac{2}{\tau_c}}\eta(t) \ ,$$

$$\beta = \pm\sqrt{1+\alpha}-1$$

$$\longrightarrow \quad C_{current}(t,t') = \sigma_w^2 \left[\delta(t-t') + \frac{\alpha}{2\tau_c}\, e^{-\frac{|t-t'|}{\tau_c}}\right]$$

I. Results. FPE and stationary response

The **FPE** associated to the equation for V and the current is

$$\boxed{[L_x+\frac{L_z}{k^2}+\frac{2}{k}\frac{\partial}{\partial x}(\frac{\partial}{\partial z}-\frac{\beta z}{2})]P_\beta(x,z)=-\tau_m\delta(x-\sqrt{2}\hat{H})J_\beta(z)}$$

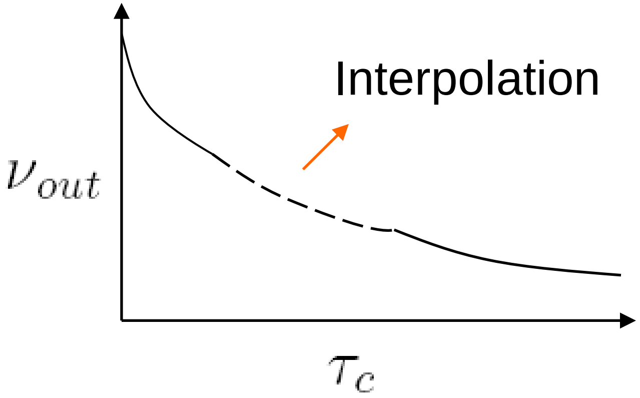
It can be solved in the **long correlation time limit**

$$\boxed{\nu_{out}=\nu_0+\frac{C}{\tau_c}}$$

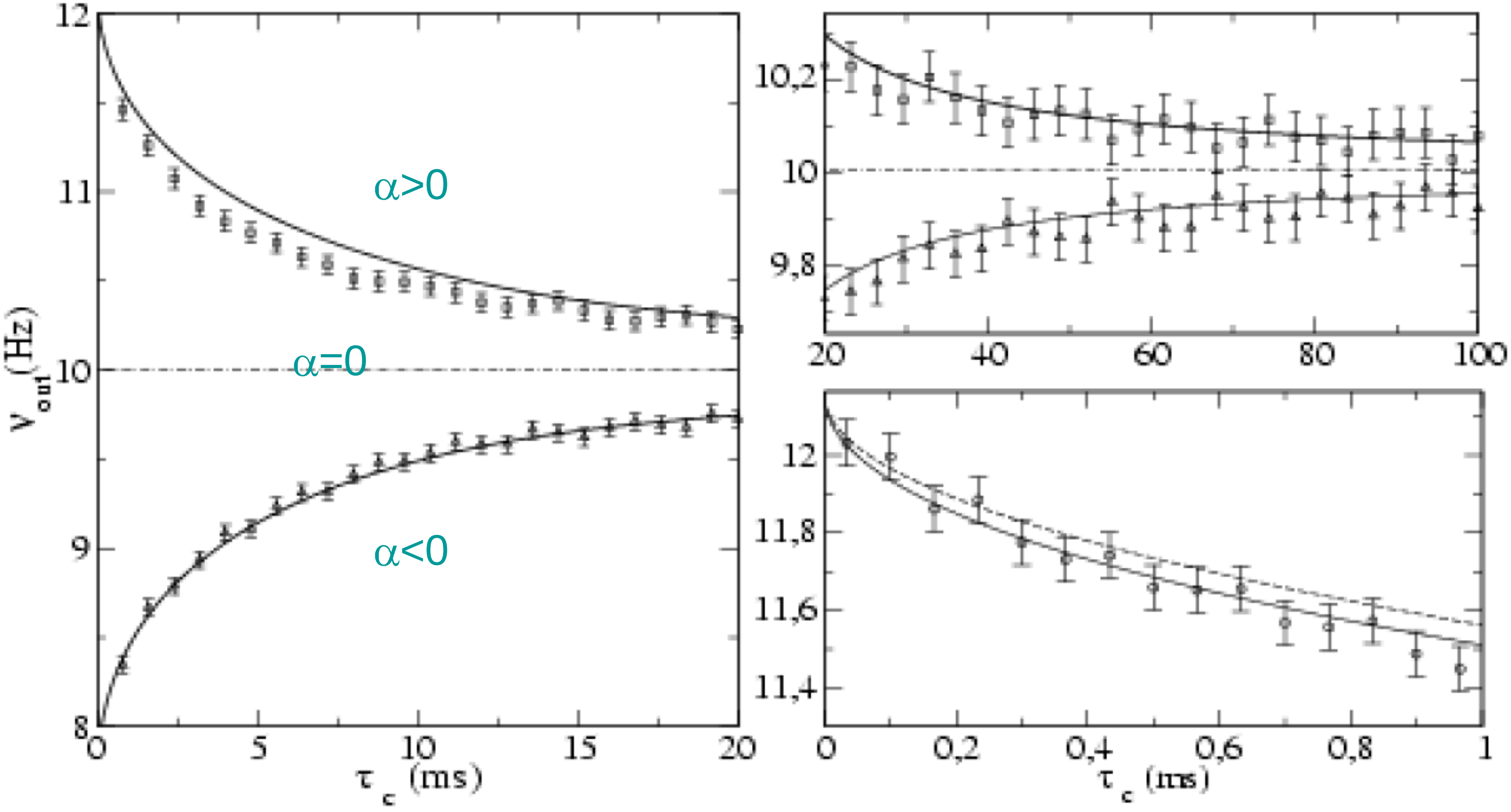
A similar FPE is solved in the **short correlation time limit**

$$\boxed{\nu_{out}=\nu_{eff}-\alpha\sqrt{\tau_c\tau_m}\nu_0^2R(\hat{\Theta})}$$

$$\sigma_{eff}^2=\sigma_w^2(1+\alpha)$$



I. Results. Stationary rate as a function of τ_c



I. Results. Non-stationary response.
Fast responses predicted by the FPE

The **instantaneous firing rate** of the neuron is exactly

$$\nu_{out}(t) = -\frac{\sigma_w^2(t)}{2}\frac{\partial}{\partial V}\int_{-\infty}^{\infty} dw P(V,w,t)|_{V=\Theta}$$

Changing $\sigma_w^2(t)$ will procude an instantaneous change in the rate

When the **correlation time** becomes **zero**, it can be expressed as

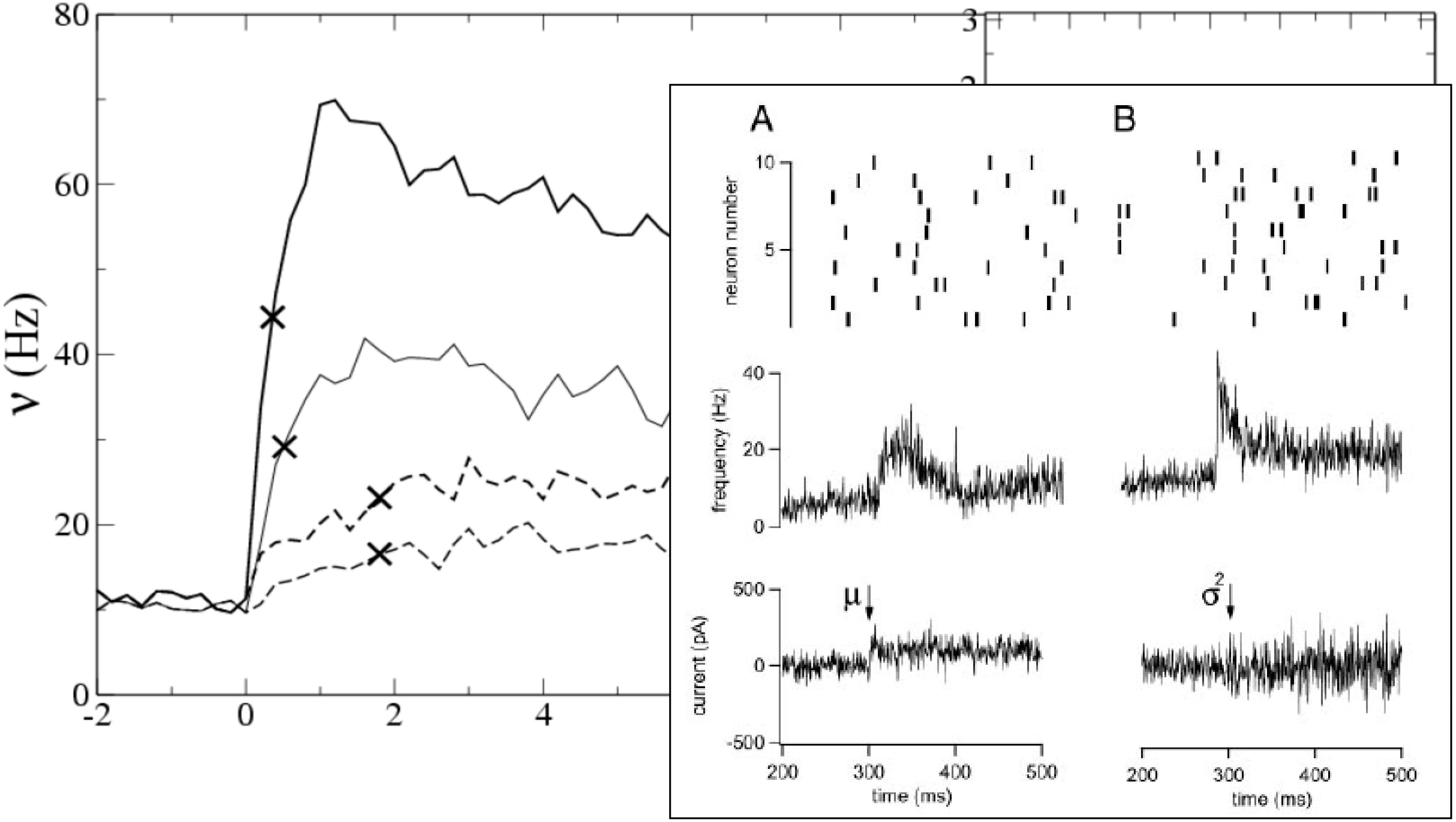
$$\nu_{out}(t) = -\frac{\sigma_{eff}^2(t)}{2}\frac{\partial}{\partial V}\int dw P(V,w,t)|_{V=\Theta}$$

$$\sigma_{eff}^2 = \sigma_w^2(1+\alpha)$$

Changing it will procude an instantaneous change in the rate

For short enough correlation times, the response has also to be very fast!

I. Results. Rapid response to instantaneous changes of α

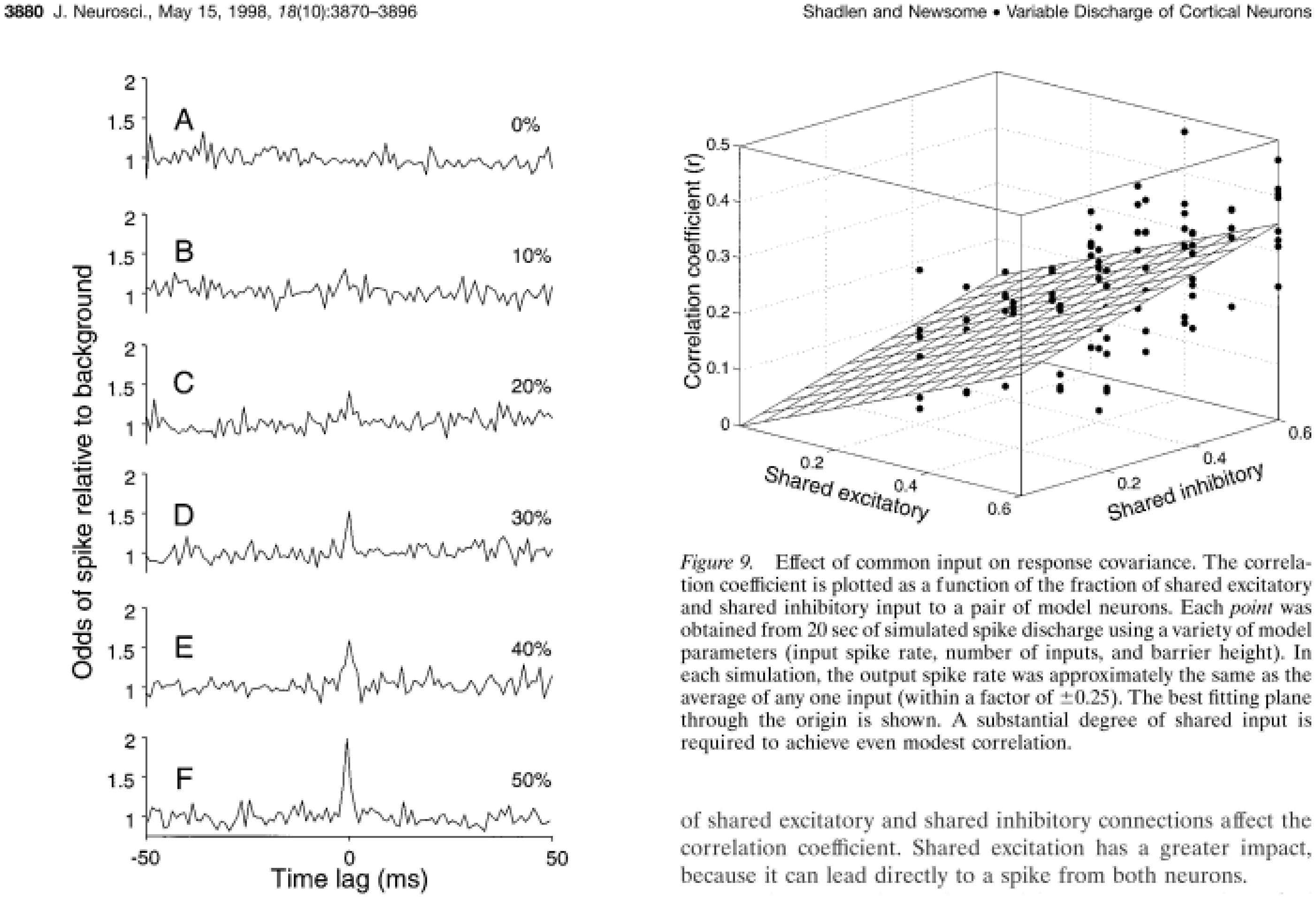


Silberberg et al, 2004

I. … in conclusion

1. We have described the statistical properties of a current that considers the acitivity of many correlated neurons.
2. This current has been generated using an auxiliary OU process.
3. The associated FPE to this current and to an IF neuron has been solved in the limits of short and long correlation times.
4. These solutions predict the modulation of neuronal resposes to variations of the parameters defining the correlated activity.
5. Changing the correlation magnitude of pre-synaptic populations produces a very fast increase of the output firng rate.

I. Weak effects of correlations on firing rate?



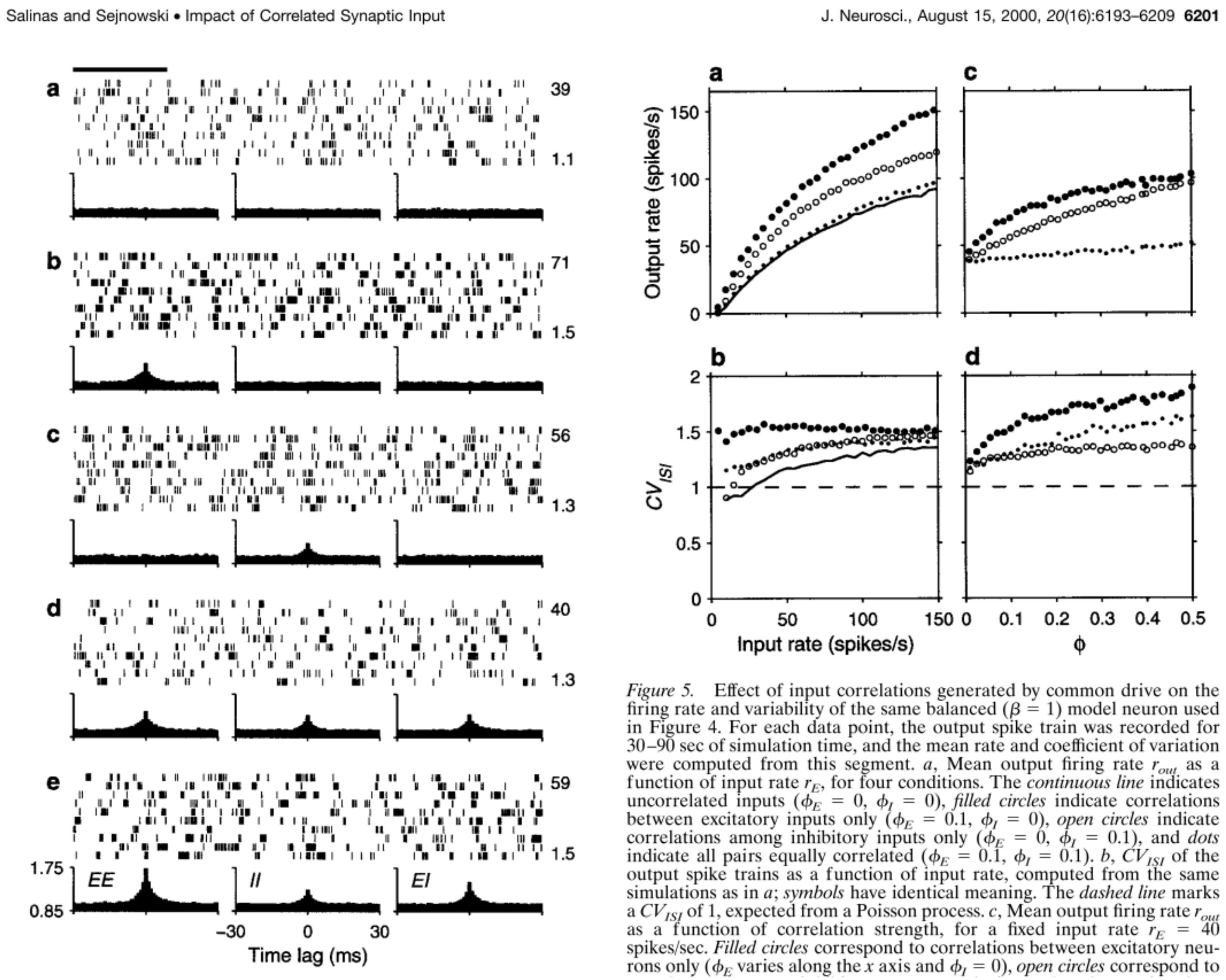
3880 J. Neurosci., May 15, 1998, *18*(10):3870–3896

Shadlen and Newsome • Variable Discharge of Cortical Neurons

Figure 9. Effect of common input on response covariance. The correlation coefficient is plotted as a function of the fraction of shared excitatory and shared inhibitory input to a pair of model neurons. Each *point* was obtained from 20 sec of simulated spike discharge using a variety of model parameters (input spike rate, number of inputs, and barrier height). In each simulation, the output spike rate was approximately the same as the average of any one input (within a factor of ±0.25). The best fitting plane through the origin is shown. A substantial degree of shared input is required to achieve even modest correlation.

of shared excitatory and shared inhibitory connections affect the correlation coefficient. Shared excitation has a greater impact, because it can lead directly to a spike from both neurons.

I. Strong effects correlations on rate and CV



Salinas and Sejnowski • Impact of Correlated Synaptic Input

J. Neurosci., August 15, 2000, *20*(16):6193–6209 **6201**

Figure 5. Effect of input correlations generated by common drive on the firing rate and variability of the same balanced ($\beta = 1$) model neuron used in Figure 4. For each data point, the output spike train was recorded for 30–90 sec of simulation time, and the mean rate and coefficient of variation were computed from this segment. *a*, Mean output firing rate r_{out} as a function of input rate r_E, for four conditions. The *continuous line* indicates uncorrelated inputs ($\phi_E = 0$, $\phi_I = 0$), *filled circles* indicate correlations between excitatory inputs only ($\phi_E = 0.1$, $\phi_I = 0$), *open circles* indicate correlations among inhibitory inputs only ($\phi_E = 0$, $\phi_I = 0.1$), and *dots* indicate all pairs equally correlated ($\phi_E = 0.1$, $\phi_I = 0.1$). *b*, CV_{ISI} of the output spike trains as a function of input rate, computed from the same simulations as in *a*; *symbols* have identical meaning. The *dashed line* marks a CV_{ISI} of 1, expected from a Poisson process. *c*, Mean output firing rate r_{out} as a function of correlation strength, for a fixed input rate $r_E = 40$ spikes/sec. *Filled circles* correspond to correlations between excitatory neurons only (ϕ_E varies along the *x* axis and $\phi_I = 0$), *open circles* correspond to

I. Weak effects of correlations on firing rate?

$$
\begin{aligned}
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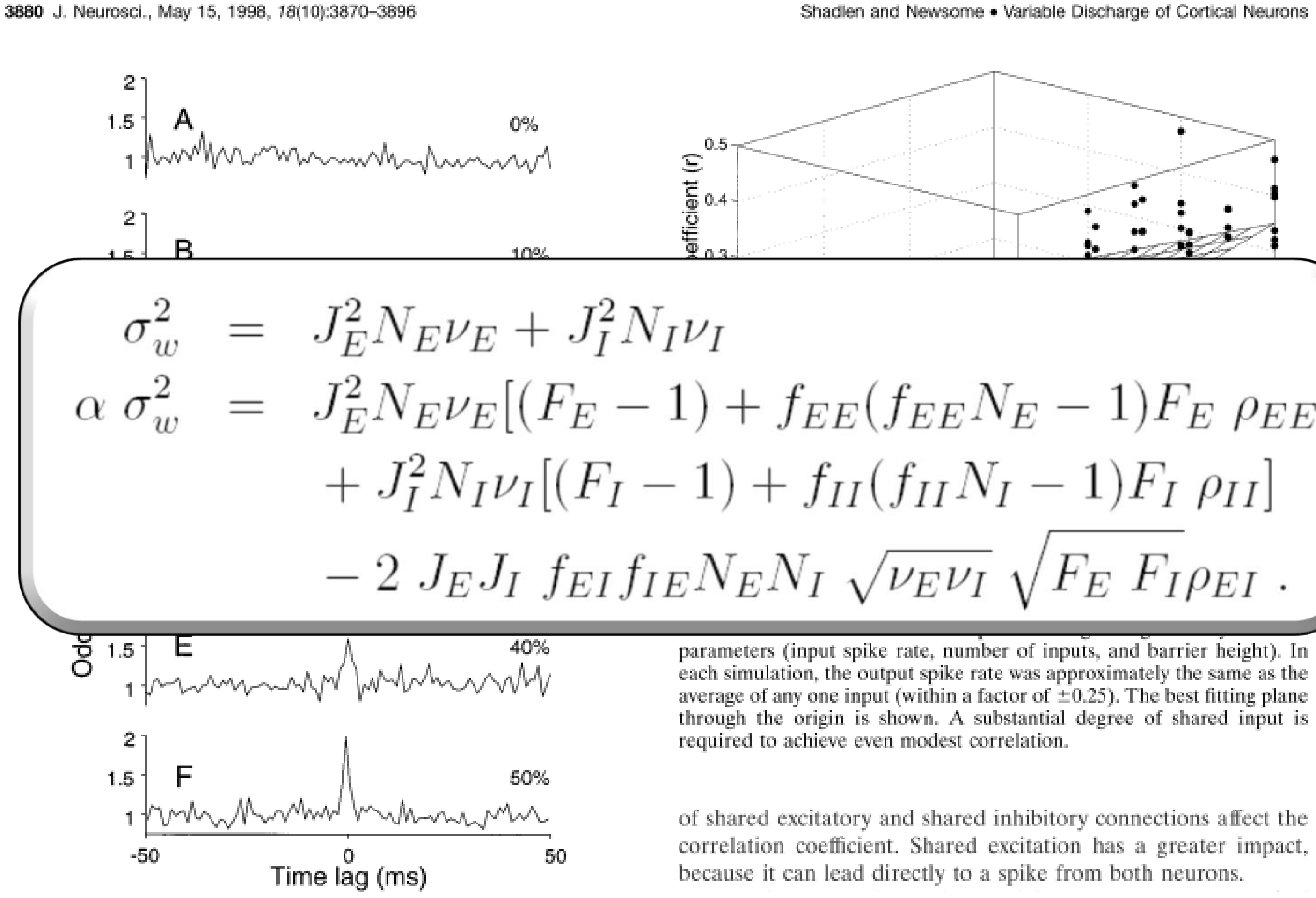
&\quad + J_I^2 N_I \nu_I [(F_I - 1) + f_{II}(f_{II} N_I - 1) F_I\, \rho_{II}] \\
&\quad - 2\, J_E J_I\, f_{EI} f_{IE} N_E N_I\, \sqrt{\nu_E \nu_I}\, \sqrt{F_E\, F_I \rho_{EI}}\ .
\end{aligned}
$$



3880 J. Neurosci., May 15, 1998, 18(10):3870–3896

Shadlen and Newsome • Variable Discharge of Cortical Neurons

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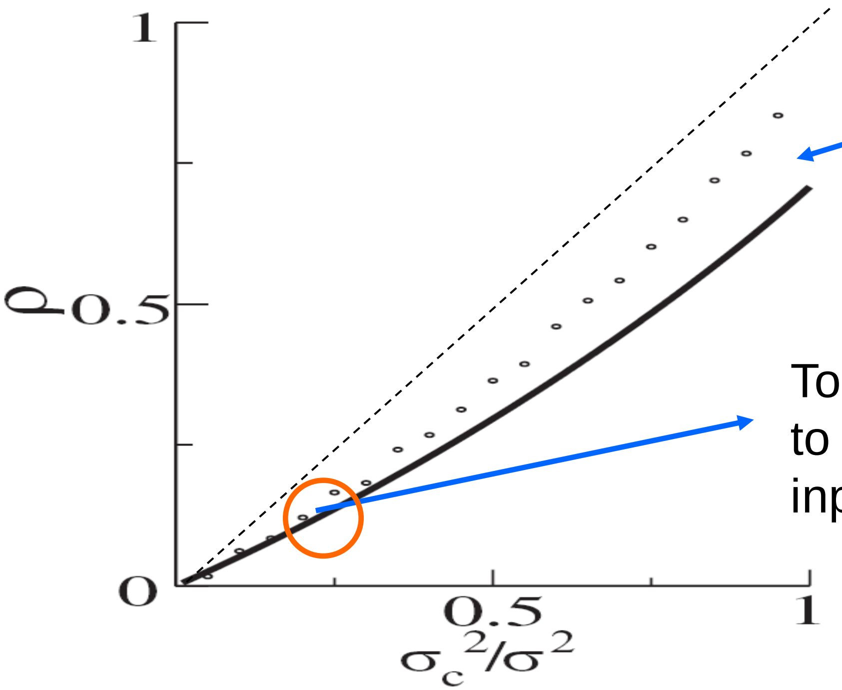
- Open questions

Correlation coefficient: $\rho = \frac{2\int C(s)ds}{F_N \nu}$

correlation

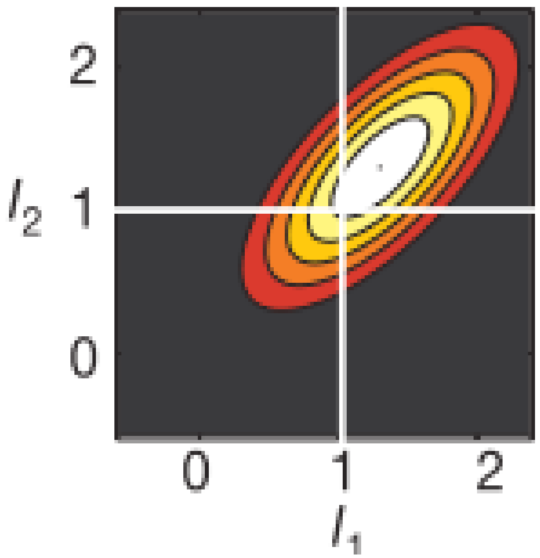


Slow rise with slope smaller than 1

To get ρ**=0.1**, neurons need to share around **20%** of their input variability

input correlation, or
fraction of common noise

$$\sigma^2 = \sigma_{ind}^2 + \sigma_c^2$$

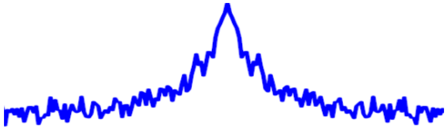


The correlation coefficient of the output of a pair of non-linear rate neurons receiving correlated Gaussian noise is bounded by the correlation in the input

Lancaster, Biometrika, 1957

Low fraction of common noise

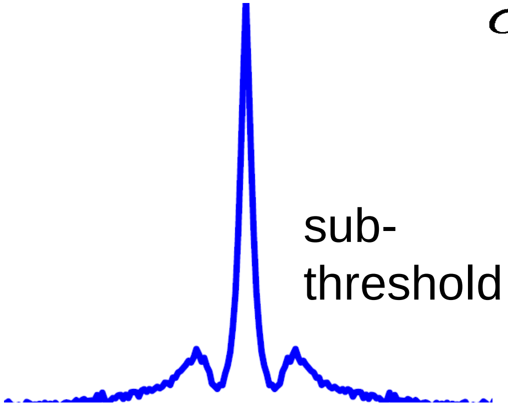
$$\frac{\sigma_c^{\ 2}}{\sigma^2} << 1$$

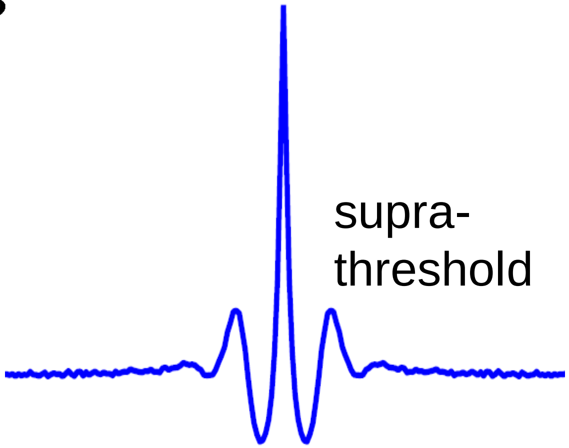


- -A **single peak** in both sub- and supra-threshold regimes
- -**Width** of the **peak** is approx. τ_s

Large fraction of common noise

$$\frac{\sigma_c^{\ 2}}{\sigma^2} \approx 1$$





- -**Damped oscillatory profile** in both regimes
- -**Width** is not simply related to τ_s

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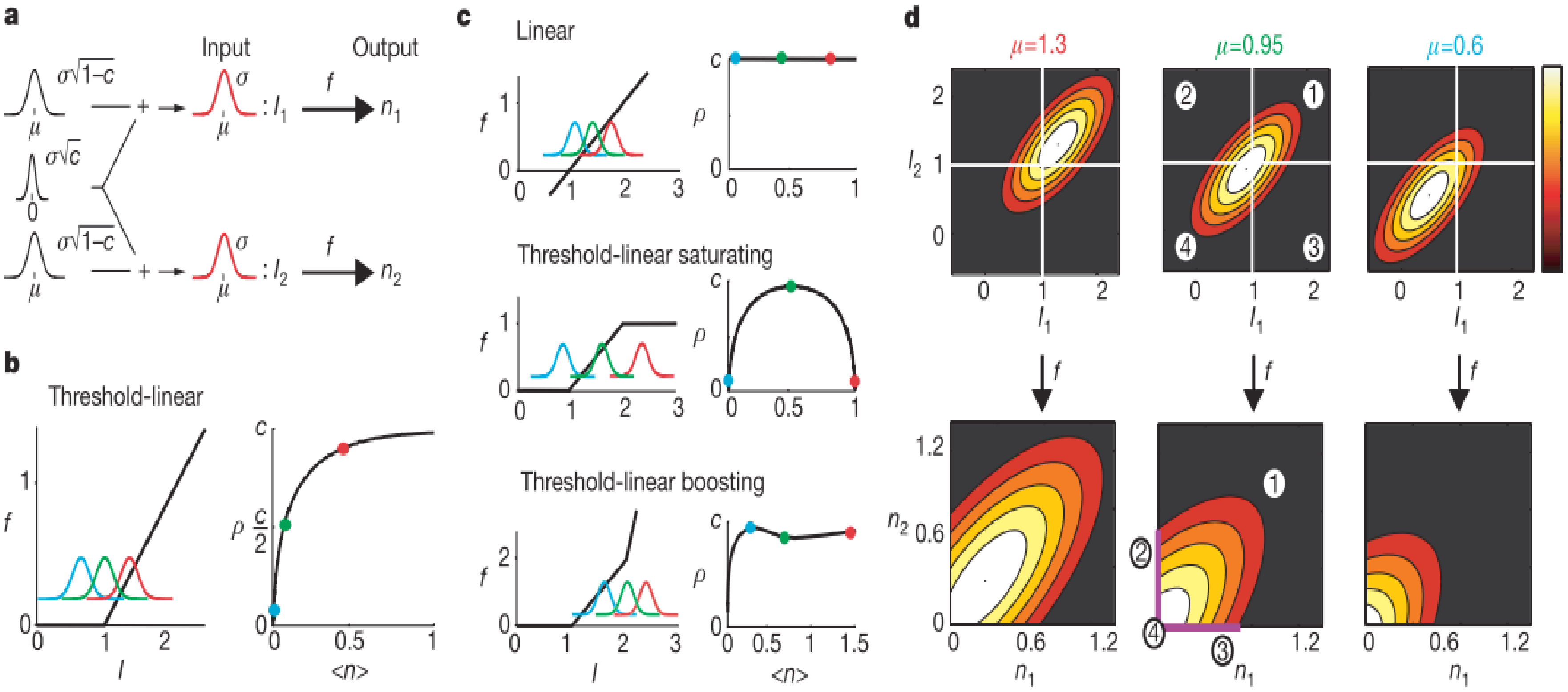
- Open questions



Output correlation increases with output firing rate

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- Open questions

Relationship Between Simulated Common Synaptic Input and Discharge Synchrony in Cat Spinal Motoneurons

MARC D. BINDER AND RANDALL K. POWERS
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Network: Comput. Neural Syst. **14** (2003) 747–763 PII: S0954-898X(03)67200-3

Influence of membrane properties on spike synchronization in neurons: theory and experiments

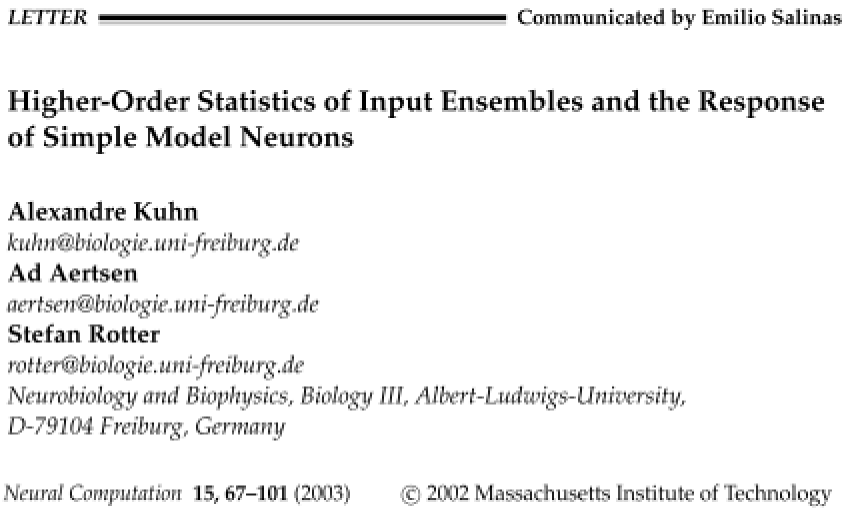
Gytis Svirskis[1,2,3] **and Jørn Hounsgaard**[3]

3646 • The Journal of Neuroscience, April 5, 2006 • 26(14):3646–3655

Behavioral/Systems/Cognitive

Correlation-Induced Synchronization of Oscillations in Olfactory Bulb Neurons

Roberto F. Galán,[1,3]* **Nicolas Fourcaud-Trocmé,**[2,3]* **G. Bard Ermentrout,**[2,3] **and Nathaniel N. Urban**[1,3]
[1]Department of Biological Sciences, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, [2]Department of Mathematics, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, and [3]Center for the Neural Basis of Cognition, Pittsburgh, Pennsylvania 15213



LETTER Communicated by Emilio Salinas

Higher-Order Statistics of Input Ensembles and the Response of Simple Model Neurons

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Neural Computation **15, 67–101** (2003) © 2002 Massachusetts Institute of Technology

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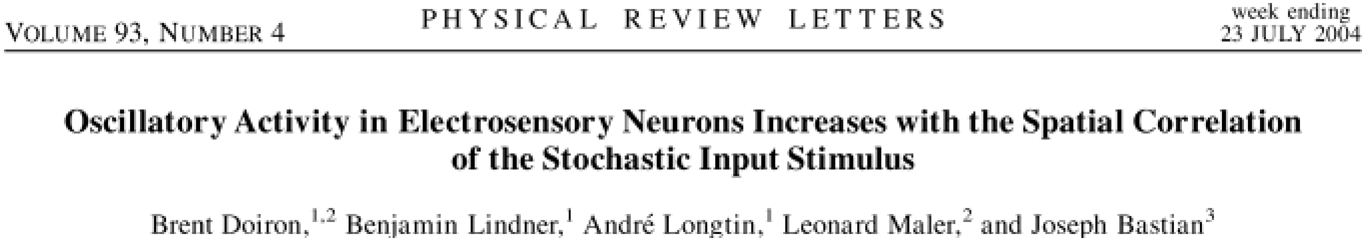
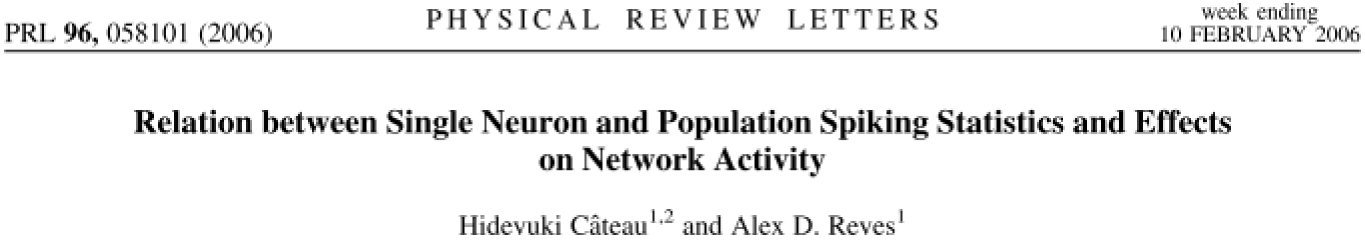


VOLUME 93, NUMBER 4 PHYSICAL REVIEW LETTERS week ending 23 JULY 2004

Oscillatory Activity in Electrosensory Neurons Increases with the Spatial Correlation of the Stochastic Input Stimulus

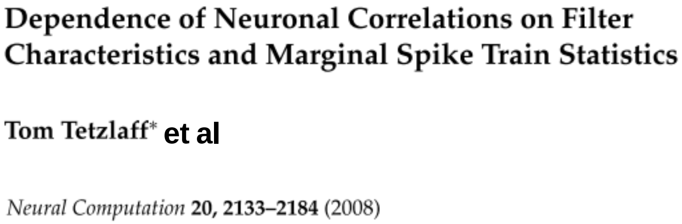
Brent Doiron,[1,2] Benjamin Lindner,[1] André Longtin,[1] Leonard Maler,[2] and Joseph Bastian[3]



PRL **96,** 058101 (2006) PHYSICAL REVIEW LETTERS week ending 10 FEBRUARY 2006

Relation between Single Neuron and Population Spiking Statistics and Effects on Network Activity

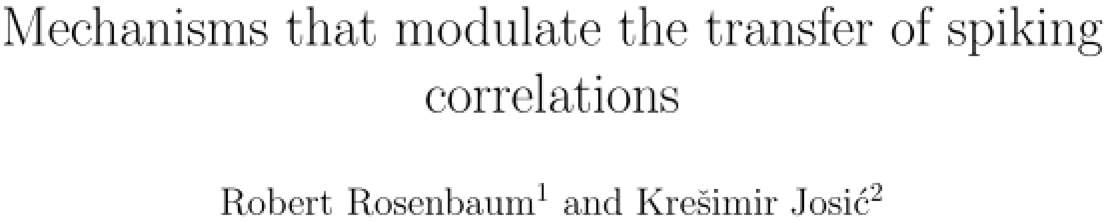
Hideyuki Câteau[1,2] and Alex D. Reyes[1]



Dependence of Neuronal Correlations on Filter Characteristics and Marginal Spike Train Statistics

Tom Tetzlaff* **et al**

Neural Computation **20, 2133–2184** (2008)

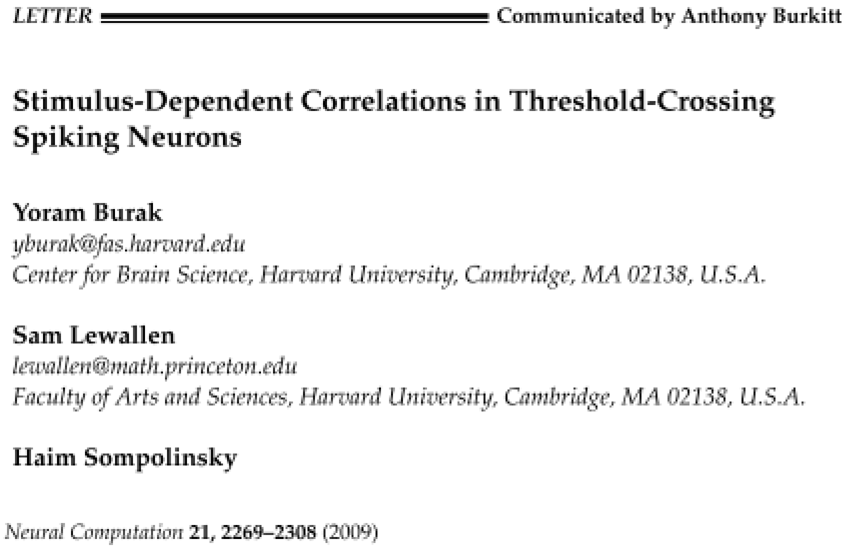


Mechanisms that modulate the transfer of spiking correlations

Robert Rosenbaum[1] and Krešimir Josić[2]

NEURAL
COMPUTATION

May 2011, Vol. 23, N. 5, Pages 1261-1305



LETTER **Communicated by Anthony Burkitt**

Stimulus-Dependent Correlations in Threshold-Crossing Spiking Neurons

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Haim Sompolinsky

Neural Computation **21, 2269–2308** (2009)

PRL **104,** 058102 (2010) PHYSICAL REVIEW LETTERS week ending 5 FEBRUARY 2010

Correlations and Synchrony in Threshold Neuron Models

Tatjana Tchumatchenko,[1,2] Aleksey Malyshev,[3,4] Theo Geisel,[1] Maxim Volgushev,[3,4,5] and Fred Wolf[1]

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- Open questions

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Open questions

- The Fokker-Planck equation (FPE) for a pair of correlated neurons remains unsolved exactly for all limits, except for one case (however, very good approximations are available in some limits, as described in this tutorial)

- How correlation transfer operates in more complex neuronal models (e.g., Hodgkin & Huxley) is not known

- How correlation transfer depends on reciprocal connections is largely unknown (but await to the 2nd part of the tutorial)

- The relationship between correlations and information in a pair of neurons remains unexplored